Paper for Presentation at 38th European RSA, Vienna, 28 Aug to 1 Sept 1998

AREAS, NODES AND NETWORKS : Some Analytical Considerations

by John R. Roy
ETUDES (Environmental, Transport and Urban DEvelopment Studies), Mallacoota, Victoria, 3892, Australia

Abstract :

In spatial interaction modelling, trips between origins and destinations within the same areal zone have a predominant influence on both the value of the gravity parameter and on the associated pattern of flows. Despite this, the relevant highly sensitive intrazonal impedance values are usually based on approximate average intrazonal distances or times. This situation has been identified in the literature as the 'self potential' problem. In this paper, integration over continuous space within the origin-destination zones is applied to not only compute the intrazonal flows more accurately, but also to evaluate the interzonal flows along shortest path routes meeting the interzonal links at efficient intermediate points. In particular, this general approach permits more accurate corrections to the conventional model, allowing, for instance, the usual approximations in determining average trip length to contiguous zones to be overcome. In addition, all jobs can be taken as concentrated at zonal centroids which are also nodes of the transport network. Alternatively, for outer zones, jobs may be taken as dispersed throughout the zonal area. This latter option reflects the greater dispersion of service jobs within suburban areas. The eventual aim is to develop practical 'rules of thumb' for correcting the conventional analysis.

Flows between areal zones and several facility nodes can occur along plausible alternative multi-destination paths, rather than via one 'abstract' interzonal path to one destination, as usually considered in conventional spatial interaction models. Such destination/path choice is easy to handle in the relatively uncongested conditions characterizing off-peak discretionary travel. This paper examines facility choice via alternative round trip routes, attempting to discern the influence of 'intervening opportunities' on the potential for multi-stop trips without having to fully identify the actual trip chain. Such intervening opportunities can only be properly considered along the alternative paths of the actual network, as specified above.

INTRODUCTION

In characterization of spatial interaction in urban areas, certain key assumptions are usually introduced for representing in space the demand entities, the supply entities and the networks via which these entities interact. These assumptions include (i) the subdivision of the urban area into discrete zones, (ii) the concentration of all demand activity or supply activity within a zone each at one discrete internal node and (iii) for trip distribution, all travel occurring along one abstract interzonal path connecting the demand node in the origin zone with one supply node in the destination zone. Although we are often 'stuck with' assumption (i) due to data availability, assumption (ii) may cause significant errors. Whereas many destination facilities can reasonably be taken to be concentrated at discrete nodes in space, the demand of households is dispersed throughout each zone. In addition, the increased frequency and dispersion of service jobs and home-based jobs in outer areas create considerable dispersion on the supply side. Finally, the restriction (iii) of travel between a zone and facility to be along the one (usually abstract) path often implies 'averaging' of routes of quite different hierarchies, as well as preventing consideration of different groupings of intervening opportunities, as potential destinations in multi stop trips along the alternative paths.

Of course, it is no accident that the above approximations have been part of practical analysis. Conventional interzonal trip distribution models are difficult enough to solve without the introduction of continuous geometry into the irregularly shaped zones for which data is usually available (eg. municipality or census tract). Also, alternative paths are usually considered in a separate trip assignment step, rather than being integrated within the trip distribution process. In this paper, there is as yet no attempt to formulate a model based on continuous geometry and multiple paths to directly handle spatial interaction in an actual urban area. Instead, the approach is to create some idealized urban forms which lend themselves to integration over continuous space and to compare the results of the new continuous geometry approach with those of the conventional procedure. Thereby, the planner should be able to identify situations where the conventional method may produce significant errors, locate the main sources and magnitudes of these errors and appropriately correct the conventional analysis. In the space of this one paper, it is only possible to characterize some types of continuous geometrical distributions of demand and supply activities. However, a quite general framework is set up which should help future researchers devise further cases of practical interest.

It had been realized for many years that the results of trip distribution models are particularly sensitive to the values of average trip times or distances chosen for the *intrazonal* trips. A consideration of these issues has been classified by Bröcker (1989) as ‘the self-potential problem’. Bröcker starts with the classical potential formula, which is also derivable from the origin-constrained gravity model. He makes three important advances on previous work (i) allowing for the increased probability of choice of a node when situated close to that node *within* a zone, (ii) transforming the potentials such that they are well-behaved at and approaching the upper limit $\beta\rightarrow\infty$ of the gravity parameter and (iii) providing upper and lower bounds on potential for $\beta=0$ and $\beta\rightarrow\infty$ respectively. At the same time, there are several points not yet considered in Bröcker’s work, including (i) corrections to the potential of zones directly contiguous with the origin zone, (ii) the consideration of some zones within which the destination opportunities are dispersed rather than concentrated at one node, (iii) the existence of variable residential or employment density within zones and (iv) an adaptation of the approach to doubly-constrained journey-to-work models.

A later paper by Frost and Spence (1995), which unfortunately does not quote Bröcker’s paper, despite appearing in the same journal, has a more empirical approach, mentioning some points not covered by Bröcker. For instance, rather than integrating over the circle with Bröcker’s general negative exponential impedance function, they merely test a linear inverse power function over a circle divided into 5 annuli. However, they do test variable housing density within the circle, as well as mentioning the increase in average trip length when opportunities are dispersed within the circle. Above all, they indicate that assumptions on self-potential can significantly affect the distribution of economic activity potential in the UK. In other words, these questions are not just of theoretical interest, but can have a strong influence on empirical analysis.

In attempting to rise to these challenges in the current paper, the entire trip distribution analysis is performed using continuous geometry, identifying further issues in addition to the use of corrected self-potential values. At this stage, the analysis concentrates on journey-to-work models, where trip generation rates are known à priori. Whereas, housing and job densities are permitted to vary *between* zones, they are generally taken as *constant* within each zone. However, the consequences of this assumption are tested for two simple cases, allowing residential or employment density to increase linearly towards the centre from a fixed outer value. Whereas the estimation problem for the doubly-constrained gravity model is expressed in general terms over continuous space following on from Angel and Hyman (1972), the actual results are given for the lower bound $\beta\rightarrow\infty$ and the upper bound $\beta=0$ of this strictly concave problem. Several interesting new results are derived in the Appendices, especially for *interzonal* travel with continuous geometry in the travel-cost-minimizing case of $\beta\rightarrow\infty$. Although some implications for modification of the conventional analysis are provided, further derivations and testing, using the same general scheme, are required to form more detailed conclusions.

If the conventional trip distribution analysis is extended by allowing more than one path for journeys starting and ending in any zone and passing several **potential destinations**, an entropy formulation given exogenous trip speeds is quite simple. This path-based approach will extend the combined gravity/intervening opportunities model introduced by Gonçalves and Ulysséa-Neto (1993) and somewhat refined in Roy (1993). After all, the opportunities are physically located along the actual paths of the network, rather than along the single abstract path generally assumed in spatial interaction models. Whereas for journey-to-work analysis, the existence of intervening opportunities between the origin and the chosen destination may reduce the probability of choosing that destination, the occurrence of intervening services on paths exiting from and returning to any origin may conceivably increase the attractiveness of that path. In other words, such 'opportunity rich' paths with their associated **destination chains** would have a greater scope for being part of multi-stop trips. The above principles are used to formulate an origin-constrained retail/service demand model.

The first section of the paper will address journey-to-work gravity models based on continuous geometry, as a pre-condition to correction of the conventional discrete geometry models. The shorter final section develops a multi-path trip distribution model, which contains both path-related gravity components, as well as the identification of intervening opportunities as potential stops along such paths.

CONTINUOUS GEOMETRY JOURNEY-TO-WORK MODELS

A General Formulation for Continuous Circular Zones

The Basic Mathematics In the path-breaking paper of Angel and Hyman (1972), the general problem of specification of the doubly-constrained gravity model for continuous geometry was formulated. In Roy(1997), the same approach was used to enhance retail gravity models with budget constraints, based on specifying as continuous the amount of goods or services consumed by a customer in any one trip. Otherwise, Angel and Hyman's work seems hardly to have been quoted during the past 25 years. Yet, a rigorous continuous geometry formulation should *à priori* be the basis for not only 'the self potential problem', but also for considering the further approximations introduced into the conventional analysis by discretization of continuous space. As an illustration of Angel and Hyman's formulation, we consider that our region consists of a number of distinct circular zones connected by transport links. The residential density within each circular zone is taken as constant and travel occurs within each zone along shortest paths on a radial/circumferential network. All jobs are taken as concentrated at the centre of each zone (see Appendix for relaxation of this assumption). Let $T_{ij}(r,\theta)$ be the intensity of travel from an origin at any point (r,θ) of origin zone i to work at the centre of destination zone j. If for illustration, each zone is taken to have the same residential density and the same radius R, the continuous doubly-constrained problem for calibration can be specified as

$$Z = {}_{\max} - \sum_{ij} \int_0^{2\pi} \int_0^R T_{ij}(r,\theta)\,[\log T_{ij}(r,\theta)-1]\, r\, dr\, d\theta \; + \; \sum_i \lambda_i\,[O - \sum_j \int_0^{2\pi} \int_0^R T_{ij}(r,\theta)\, r\, dr\, d\theta]$$

$$+ \; \sum_j \eta_j\,[D - \sum_i \int_0^{2\pi} \int_0^R T_{ij}(r,\theta)\, r\, dr\, d\theta] + \beta\,[T\, d^* - \sum_{ij} \int_0^{2\pi} \int_0^R T_{ij}(r,\theta)\, d_{ij}(r,\theta)\, r\, dr\, d\theta] \quad (1)$$

where O and D are the number of workers and jobs respectively in each zone, $d_{ij}(r,\theta)$ is the shortest path travel distance from any point (r,θ) in zone i to jobs at the centre of zone j. Note that, as shown in the Appendix, for $0 \leq \theta \leq 2$, $d_{ij}(r,\theta)=l_{ij} + [r\theta + (R-r)] + R$ on the radial/ circumferential network, where l_{ij} is the shortest path distance between the peripheries of zones i and j . For $\theta>2$, travel to exit zone i on the way to zone j occurs purely radially through the centre of i, and we have $d_{ij}(r,\theta)=l_{ij} + [r + R] + R$.

As shown by Angel and Hyman (1972), the above problem type can be solved by initially differentiating the Lagrangian under the integral sign and equating the result to zero, after which the $T_{ij}(r,\theta)$ expression is substituted into the constraints to yield expressions for the unknown Lagrange multipliers λ_i, η_j and β. These can be solved for iteratively, analogously as for the conventional discrete doubly-constrained model. Note that, the continuous problem retains the strictly concave and strictly positive nature of the discrete problem, implying that the solution for the flows is unique and positive. Although, we could have straightforwardly solved the above problem for a wide range of values of the gravity parameter β, this paper specializes the comparisons to the lower bound travel-distance-minimizing case where $\beta \rightarrow \infty$, and to the upper bound travel-distance-indifferent case $\beta=0$. As shown in the Appendices, analytical solutions are possible for these limiting cases, permitting a greater transparency compared with numerical solutions.

General Criteria for Interpretation of the Results In lieu of a continuous analysis of real urban systems containing irregular zones, the continuous formulation is being used in this paper with simple geometry as a means to potentially improve the solution of the conventional models. It is well known that the gravity parameter β is roughly proportional to the reciprocal of the average trip length d_{ave} over the entire system. For instance, in our retail model LAIRD, we have found that $\beta \approx 1.3 / d_{ave}$ provides a reliable starting value for the iterative calibration procedure in most applications. Thus, if the continuous analysis reveals systematic errors in the average trip length d_{ave} , a possible strategy may be merely to correct β inversely, that is, if d_{ave} is expected to be underestimated by say x% by the conventional model, we may increase β by x% compared with its calibrated value. However, such an approach would tend tend to rather evenly weight the corrections to both the intrazonal and interzonal flows, rather than giving appropriate extra consideration to the larger potential errors in the intrazonal trip distances and corresponding flows. Thus, it is suggested that β be retained at its calibrated value, but that corrections be made directly to both the intrazonal and interzonal impedance values [exp - β d_{ij}], confining the latter corrections usually to the distances to contiguous zones. Finally, the gravity model can be re-run with the revised impedance values. Thus, the results below give not only the errors in overall average trip distance, but decompose this to errors in intrazonal vs interzonal average distances. In real problems containing a large number of zones, the analyst can distribute the interzonal error over a chosen set of adjacent zones.

Some Specific Upper and Lower Bound Solutions

Linear City with 3 Circular Zones The city consists of three circles of unit radius with

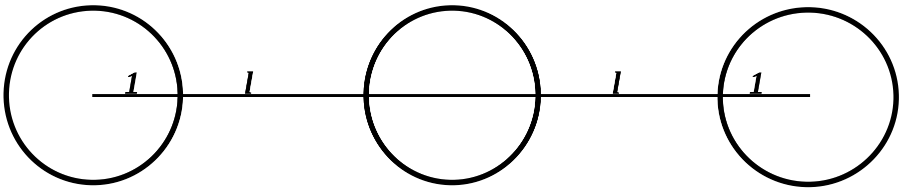


FIG. 1

their outer rims separated by a distance *l*, as seen in Fig. 1. Whereas *l* may be positive for discrete townships, it is set to zero to simulate an urban area with a central business district (CBD) and two outer sub-centres. As both housing and employment is likely to be more dense in the inner zone, we designate as $a \geq 1$ the ratio of (uniform) housing density between the central and outer zones, and $b \geq 1$ the ratio of jobs between the central and outer zones. Although the jobs in the central zone are always taken as concentrated at the CBD, the jobs in the outer zones are either taken as concentrated at the zonal centroids ($k_j=0$) or distributed uniformly throughout the zones ($k_j=1$). This latter option allows consideration of the increasing prevalence of dispersed service employment in the outer suburbs of many large cities.

A computer program was written to test the influence of variations of relative housing concentration 'a', relative employment concentration 'b' and separation '*l*' between the rims of the centres, for either concentrated outer jobs ($k_j=0$) or dispersed outer jobs ($k_j=1$). This was performed for both the lower bound solution $\beta \rightarrow \infty$ and the upper bound case $\beta=0$, using the results of the Appendices. In the lower bound special case for dispersed outer jobs ($k_j=1$) and relative job concentration 'b' greater than the relative housing concentration 'a', where some commuting occurs from the outer zone to the CBD, a closed form analytical solution did not seem possible. Instead of resorting to numerical integration, a simple heuristic was used which was conservative with respect to the comparisons being made. Whereas all jobs in the outer zones in this special case were assumed to be filled by workers living 'next door', the workers commuting to the CBD were also taken as evenly distributed over the outer zones. As seen later, this assumption was not crucial in the key comparisons.

In assessing the potential improvements of the continuous solution, it is necessary to define the basis for the conventional solution. Firstly, as we are primarily interested in *relative* errors, the analysis is confined to cases where the housing density does not vary *within* a zone and the employment density does not vary *within* an outer zone. Under this assumption, it is being hypothesized that density variations would affect the conventional result and the continuous result rather similarly. Thus, for the conventional solution it is assumed that (i) the average distance for intrazonal travel in the unit circle is (2/3) [see Appendix 1B] and (ii) the distances for interzonal travel are all centroid to centroid distances. Although the program was run for a very wide spectrum of cases, only the most interesting results are quoted in the following.

For all cases, a finite separation of the circles (l>0) reduced (or kept constant) the relative error compared to when the circles are touching (l=0). This was to be expected, as the effects of the ever present intrazonal approximations on overall average trip distance are lessened as the less sensitive interzonal travel distance is increased. For instance, average distance errors decreased in a typical case from 14% to 10% when the separation l was changed from zero to the unit radius. As a result, the following results deal only with the l=0 case, typical for urban areas. The average trip distance is defined as d_{ave} for the continuous model and d_c for the conventional case, with subscript a being added for the intrazonal part and z for the interzonal part.

Case 1 : Outer jobs concentrated (k_j=0) Housing density in central zone = 2 times housing density in outer zones (a=2) No. of jobs in inner zone = 4 times no. of jobs in each outer zone (b=4)

$\beta\rightarrow\infty$ Overall Averages d_{ave}= .8275 d_c= .8889 Error=+7.4%

Intrazonal Averages | Interzonal Averages

d_{avea}= .6482 d_{ca}= .6667 Error=+2.8% d_{avez}=1.7236 d_{cz}=2.0000 Error=+16.0%

β**=0** Overall Averages d_{ave}=1.7524 d_c=1.6111 Error=-8.1%

Intrazonal Averages | Interzonal Averages

d_{avea}= .6667 = d_{ca} Error=0% d_{avez}=2.2120 d_c=2.0000 Error=-9.6%

Case 2 : Outer jobs dispersed (k_j=1) Housing density in central zone = 2 times housing density in outer zones (a=2) No. of jobs in inner zone = 4 times no. of jobs in each outer zone (b=4)

$\beta\rightarrow\infty$ Overall Averages d_{ave}= .7070 d_c= .8889 Error=+25.7%

Intrazonal Averages | Interzonal Averages

d_{avea}= .4000 d_{ca}= .6667 Error=+66.7% d_{avez}=2.2422 d_{cz}=2.0000 Error ≈-10.5%

β**=0** Overall Averages d_{ave}=1.8403 d_c=1.6111 Error=-12.5%

Intrazonal Averages | Interzonal Averages

d_{avea}= .7321 d_{ca}= .6667 Error=-8.9% d_{avez}=2.3028 d_{cz}=2.0000 Error=-13.2%

The above two cases have residential density gradients and inner/outer job distributions which are typical for actual cities. Thus, we can use them to draw rather useful conclusions as follows :

(i) For almost all situations, the error in the conventional model is less when the outer jobs are concentrated, rather than evenly dispersed, especially for $\beta\rightarrow\infty$. For dispersed jobs, the conventional model precludes tele-commuting (or jobs 'next door'), yielding errors in intrazonal trip distances of 50 to 60%. Errors for the concentrated jobs solutions (0.0 to 16.0%) all lie within acceptable bounds.

(ii) Except for the case with the continuous heuristic solution (indicated by ≈), which is anyhow conservative, the conventional model over-estimates the average trip distances for the lower bound case ($\beta \rightarrow \infty$) and underestimates it for the upper bound case (β=0). This is due for $\beta \rightarrow \infty$ to the lack of any optimal spatial selection of workers for intrazonal vs interzonal jobs in the conventional model compared with the optimal profile of the continuous model in Case 3B of the Appendix. For β=0, the continuous model in Case 3A of the Appendix shows that the dispersed workers commute 24.2% more to exit from their zone than in the conventional model, where all exits are taken to emanate from the centroid.
(iii) For the case of $\beta \rightarrow \infty$ and concentrated outer jobs, the error in interzonal distance (16%) is greater than that for intrazonal distance (2.8%). In this case, as shown in Case 3B of the Appendix, the interzonal workers are all clustered close to the zonal exit node A rather than at the more distant centroid.
(iv) For the case of β=0, the errors in interzonal distances (9.6 to 13.2%) are greater than those for intrazonal distances (0.0 to 8.9%). This is because, when jobs are concentrated, intrazonal workers are forced to choose jobs at the centre of their zone, not being able to exercise the random choice implied by the β=0 designation.

Although results from the other 22 cases which were run could be quoted, the above case is strongly representative of the housing density variations and job concentration levels in actual cities. Also, as the relative errors did not vary widely for the other configurations, the solution above is sufficient to characterize the problem. Discussion of the implications for practical analysis of these results are postponed until after consideration of the 5-zone problem below.

Quadratic City with 5 Circular Zones Whereas the previous case typifies some coastal cities, the quadratic city in Fig.2 below is more generally representative. As before, the outer rims of each circular zone are separated by a distance *l* which is taken as zero for a contiguous urban area. When proceeding from an outer zone to another outer zone at right angles to it, shortest path travel is taken to proceed around the outer rim of the central zone. Travel between outer zones in line with each other is taken to proceed in a straight line through the CBD at the centre of the central zone. In future comparisons, it would be possible to experiment with alternative interzonal transport networks. For instance, one could have a ring road connecting the centroids of the four outer zones, which is an increasing trend in modern large cities. At this stage, we concentrate on the quadratic interzonal transport network as in Fig. 2. The other definitions are similar to those in the previous 3-zone case, with just the 'urban' *l*=0 case being considered, where each of the outer zones is tangential to the central zone. Note that, the main difference between the 3 zone and 5 zone cases is that the role of the central zone is more pivotal

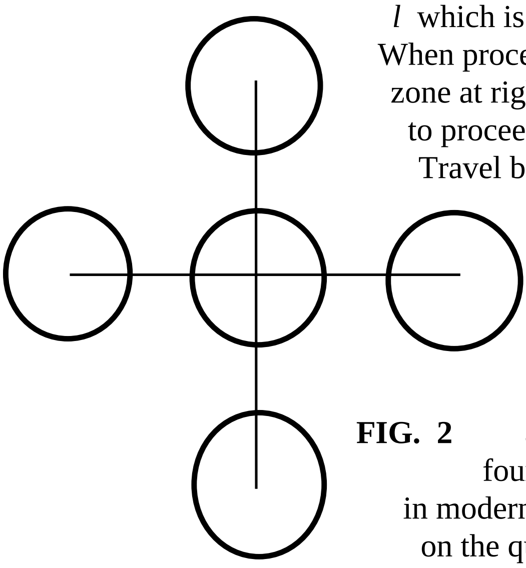


FIG. 2

for the 3 zone example, as it is accompanied by only two outer zones instead of four. The results that now follow are for the same relative densities as previously.

Case 3 : Outer jobs concentrated (k_j=0) Housing density in central zone = 2 times housing density in outer zones (a=2) No. of jobs in inner zone = 4 times no. of jobs in each outer zone (b=4)

$\beta \rightarrow \infty$ Overall Averages d_{ave}= .8059 d_c= .8889 Error=+10.3%

Intrazonal Averages Interzonal Averages

d_{avea}= .6418 d_{ca}= .6667 Error=+3.9% d_{avez}=1.6267 d_{cz}=2.0000 Error=+23.0%

β=0 Overall Averages d_{ave}=2.2768 d_c=2.1667 Error=-4.8%

Intrazonal Averages Interzonal Averages

$d_{avea=}$ .6667 = d_{ac} Error=0% d_{avez}=2.8135 d_{cz}=2.6667 Error=-5.2%

Case 4 : Outer jobs dispersed (k_j=1) Housing density in inner zone = 2 times housing density in outer zones (a=2) No. of jobs in inner zone = 4 times no. of jobs in each outer zone (b=4)

$\beta \rightarrow \infty$ Overall Averages d_{ave}= .5959 d_c= .8889 Error=+49.2%

Intrazonal Averages Interzonal Averages

d_{avea}= .2667 d_{ac}= .6667 Error=+150% d_{avez}=2.2422 d_{cz}=2.0000 Error≈-10.5%

β=0 Overall Averages d_{ave}=2.4050 d_c=2.1667 Error=-9.9%

Intrazonal Averages Interzonal Averages

d_{avea}= .7757 d_{ac}= .6667 Error=-14.1% d_{avez}=2.9481 d_{cz}=2.6667 Error=-9.6%

In comparison of the 5 zone results above with those for the 3 zone example in ***Cases 1 and 2***, it is seen that not only are the directions of the changes identical, but their magnitudes do not vary markedly. The only real exception is in the $\beta \rightarrow \infty$ run for dispersed outer jobs (***Case 4***), where the error for intrazonal flows is +150%, with the average overall error still being 49%. This is due to the greater number of dispersed jobs in the 4 outer zones, many of which are filled by tele-commuters or workers who live ‘next door’. Otherwise, the comparisons are similar to those below ***Cases 1 and 2*** and will not be duplicated here.

Implications of the Results for Practical Analysis

The provision of the above test cases and the corresponding relative errors of the conventional vs the continuous analysis must always just be regarded as a general guide for correcting the results of a conventional model. The analyst should not accept the recommendations as a rigid formula - merely as a set of guidelines. Further continuous formulations of the type developed in this paper will need to be developed to cover a wider set of potential applications.

One of the most interesting aspects of the results is the overestimation by the conventional model of the average distances for the $\beta \rightarrow \infty$ case and its underestimation for β=0. In the former, the conventional model fails to allow for the fact that the most cost-efficient pattern will have interzonal origins allocated to the part of the zone closest to the exit node rather than to the centroid. Also, for intrazonal travel in zones with dispersed outer jobs, the conventional approach neglects tele-commuting and jobs ‘next door’. Of course, the conventional models were developed at a time before the large scale suburbanization of service employment. For β=0, the radius of the circle used conventionally underestimates the average distance travelled, either from randomly dispersed worker origins or to possible randomly dispersed outer jobs, as illustrated in Cases 1A and 3A of the Appendix. Of course, this means that there is some 'magical' intermediate value of β for which the error disappears. But, it is best not to rely on such 'miracles' in practice. One can adopt the following general strategy.

Firstly, having a knowledge of the actual average trip length d_{ave}, it is possible to replace the β parameter, dimensioned as $(\text{distance})^{-1}$, by an 'equivalent' dimensionless gravity power 'n' via the relation

$$n = \beta\, d_{ave} / \log d_{ave} \qquad (2)$$

When n exceeds the 'Newtonian' value of 2, it can be considered that we are approaching the area of applicability of the $\beta \to \infty$ guidelines, and for $n<2$ we give most weight to $\beta=0$. Secondly, as shown in the Appendix, Case 2B for large n, an increase of residential density as one approaches the CBD can reduce the intrazonal travel from (2/3) the equivalent zonal radius to as low as (1/2) of this radius. However, as this density change is usually spread over several zones as one moves outwards, it is likely that only the zone which encloses the CBD should have its trip distance reduced, and perhaps to no lower than 0.6 times the zonal 'radius'. Conversely, for small n and dispersed outer jobs, as seen in Case 1A of the Appendix, the average distance is up to .994 times the radius, rather than (2/3). For the CBD zone this could be reduced to say .85 to allow for the effect of increased density towards the centre.

When the suburban jobs are strongly **concentrated** at a single point of their zone, we should be guided by Cases 1 and 3. This implies the following :

n>2 The average **intrazonal** distance should be reduced by 3 to 4% from .6667, that is to about **.64** times the equivalent zonal radius. If significant decreases in residential density occur from the CBD to the outer rim of the CBD zone, this value could be reduced to as low as **.57** times the radius. For **interzonal** travel to contiguous zones, the average centroid to centroid trip distances should be reduced by **10 to 20%** , especially if the 'gravity index' n is 3 or 4. For values of n closer to 2, this reduction could be say about **5-10%**.

n<2 For the **intrazonal** cases, the average distances should be kept at about **(2/3)** of the radius, except for the CBD zone when the residential density gradient is significant, where it could be reduced as low as **.57** times the radius. However, for **interzonal** travel, the situation is reversed, with the average distance being **increased** by **5 to 10%**.

On the other hand, when the outer jobs are strongly **dispersed** throughout their zones, more significant changes are required, especially to outer zone trip distances, as illustrated in Cases 2 and 4.

n>2 For **intrazonal** travel to the CBD, the guidelines are as above. However, for the **outer zones**, the degree of dispersal together with the extent to which n exceeds 2 should guide the choice of average distance. The lowest feasible value seems to be about **0.3** of the radius, with **0.4 to 0.5** being more typical. Information on the level of tele-commuting will also guide this choice. For **interzonal** travel to contiguous zones, and high values of n, one could make a slight reduction, by say **5 to 10%**, to allow for skewing of worker or job choice more towards the zonal exit points, rather than at zonal centroids - this was not considered rigorously in our one heuristic solution.

n<2 For **intrazonal** travel in the outer zones, values of average distance of about **.75 to .9** the radius should be used, especially as n becomes close to unity or less, as seen from Case 1A in the Appendix. For the CBD zone, with concentrated jobs, the value could be as low as say **.57** times the radius, as seen above, to allow for the density gradient. For **interzonal** travel, the average distance to contiguous zones should be increased by about **10 to 15%** to allow for 'random' exit and entry.

From the above, it is clear that the main 'grey' area is to account for cases when the equivalent gravity index is about 2, as the recommended corrections are reversed upon approaching such a value from above vs from below. However, if one understands the reasons for the various corrections as explained above, one should be able to make reasonable estimates for such borderline situations. One of the main difficulties at such intermediate n values is that the 'optimal' choices of jobs made by workers from close to the edges of zones to just 'across the border' in adjacent zones are diluted with some rather 'random' (distance-independent) job choices of other workers which are more distant, on the average, than the centroid to centroid distances. With this caveat, it is believed that the guidelines provided give a stronger basis for appropriate corrections to the conventional than currently existing rules.

MULTI-PATH GRAVITY / INTERVENING OPPORTUNITY MODELS

As shown by Gonçalves and Ulysséa-Neto (1993) and Roy (1993), it is possible to integrate the gravity and intervening opportunities models. This allows the traditional distance-deterrence effect to be combined with the influence of the opportunities intervening between the origin and final destination. Following the introduction of the approach in the paper of Gonçalves and Ulysséa-Neto (1993), Roy (1993) pointed to a combined entropy/maximum likelihood estimation procedure, as well as giving a suggestion for avoiding potential multi-collinearity between the gravity constraint and the intervening opportunities constraint. In Roy (1993), analogies with the approach of Fotheringham (1986) were discussed, in relation to his extra term expressing the relative accessibility of adjacent 'competing' destinations. As sets of 'intervening opportunities' lie physically along alternative **paths** exiting from and finally returning to any origin, one often needs a **network-oriented** approach to capture their influence. At the same time, the identification of alternative network paths for the shoppers and the destinations passed thereby allows recognition of the **potential for trip chains**. These ideas are implemented in the following entropy formulation, which is developed for retail/service travel on relatively uncongested networks. For such travel, it is postulated that the intervening opportunities constraint will have a positive parameter in terms of the total number of opportunities on each path, in contrast to the usual negative 'gravity' parameter associated with the path travel time constraint.

Consider a set of plausible paths n_{ij} connecting each origin zone i through each destination j (and including other potential destinations) and consider that S_{ijn} trips occur along each path n_{ij} during the study interval within an overall total of S trips. We also know the number of trips O_i emanating from each origin zone i and the number of trips D_j which are registered at each destination facility j. Also, the number of intervening opportunities w_{ijn} on each path n_{ij} exiting and returning to zone i via facility j is known. Units of floorspace are appropriate intervening opportunities in a retail/services model. The intervening opportunities include the floorspace units of all potential destinations on the path n_{ij}. Finally, from surveys, one should determine the average overall trip time 't' and the average intervening opportunities 'w' which are 'consumed' per trip. Note that, in this formulation for relatively uncongested networks, it is not necessary to identify individual links along the alternative paths. The entropy maximization problem is specified as follows

$$Z \overset{max}{=} - \sum_{ijn} S_{ijn} [\log S_{ijn}/(O_i D_j) - 1] + \sum_i \lambda_i [O_i - \sum_{jn} S_{ijn}] + \sum_j \eta_j [D_j - \sum_{in} S_{ijn}] \\ + \beta [St - \sum_{ijn} S_{ijn} t_{ijn}] - \alpha [Sw - \sum_{ijn} S_{ijn} w_{ijn}] \qquad (3)$$

In this objective, it is being assumed that, whereas the probability of making a trip between an origin and a destination is inversely related to the total travel time t_{ijn} between them on any given path, it is positively related to the number of intervening opportunities - but this is an empirical question relating to increased propensity for multi-destination shopping along travel paths which pass by several alternative facilities. The sign is typically in the other direction for journey-to-work models. The maximization process yields the following expression for S_{ijn}

$$S_{ijn} = O_i A_i D_j B_j \exp (\alpha w_{ijn} - \beta d_{ijn}) \qquad (4)$$

where the 'balancing factors' $A_i = \exp - \lambda_i$ and $B_j = \exp - \eta_j$ are determined via the usual recursive relations. The above model can be solved iteratively using traditional methods for the unknown flows S_{ijn} and the unknown Lagrange multipliers α and β, which are treated as parameters in impact analysis. As the number of trip destinations D_j will not be known in impact analysis, it is necessary to express the destination balancing factors B_j in terms of attractiveness characteristics of the destinations. If this is limited to the role of the floorspace W_j alone, one can interpret B_j as a j-specific power of the floorspace W_j. Alternatively, one could perform a log-linear analysis to fit the set of ($\log B_j$) to the relative level 'consumed' of each of the set of chosen characteristics (including floorspace).

It is interesting to compare the spatial relevance of the above approach with that in the work on competing destinations by Fotheringham (1986). Our procedure assumes implicitly that one takes one of several plausible **paths exiting from and returning to the origin** via a destination j, potentially being in a position to 'sample' some of the intervening opportunities **at destinations other than j during the journey**. On the other hand, the **implicit** assumption of Fotheringham's approach is that one arrives at the primary destination with the potential to take a **'star pattern'** of trips to competing **surrounding** destinations. We may conjecture that our model relates to multi-purpose and **multi-stop** trips by car or subway from an origin which passes several potential destinations before returning. As one passes (and potentially samples) all the intervening facilities during the overall trip, only the **total number of opportunities** and the **total path time** are important, not the relative accessibility of any one destination to the origin. The approach of Fotheringham (1986) seems appropriate in cases where say one travels to a **primary destination** by car or public transport, perhaps parking there, and then potentially taking short independent trips, perhaps by bus, to adjacent 'competing' destinations, whose relative accessibility to the primary destination is the main consideration. The Central Business District would be an appropriate primary destination from which such trips could emanate.

In conclusion, we have the genesis of a **trip chaining** approach as the first step towards an **activity-based** analysis. In this 'compromise' procedure, shoppers are sampled at a common time at all destinations to avoid double-counting. Thus, we cannot and do not ask them to complete a complicated travel diary, but merely to identify their **expected path** for the total journey. They may not necessarily **stop** at all destinations passed during such a journey. However, we postulate **on the average** that the potential attractiveness of the path is proportional to the **total** number of opportunities along the path, **including** those in the destination j where the shoppers are sampled. By merely identifying the **potential for trip chains** as a major influence on the choice of any given destination, the severe combinatorial problems in the implementation of **activity-based** approaches can be avoided. The approach remains similar in principle for work-based lunchtime shopping trips, as well as for shopping journeys made on the way home from work.

References

Anderson, M., Roy, J. R. and Brotchie, J. F. (1986). 'Some aspects of the adaptability of alternative urban configurations', *Environment and Planning B,* 13, 305-318.

Angel, S. and Hyman, G.M. (1972). 'Urban spatial interaction', *Environment and Planning A*, 4, 99-118.

Bonsall, P. (1975). 'Approaches to the prediction of intra-zonal interactions', Working Paper No. 65, Institute for Transport Studies, University of Leeds.

Bröcker, J. (1989). 'How to eliminate certain defects of the potential formula', *Environment and Planning A*, 21, 817-830.

Fotheringham, A. S. (1986). 'Modelling hierarchical destination choice', *Environment and Planning A*, 18, 401-418.

Frost, M. E. and Spence, N. A. (1995). 'The rediscovery of accessibility and economic potential : the critical issue of self-potential', *Environment and Planning A*, 27, 1833-1848.

Gonçalves, M. B. and Ulysséa-Neto, I. (1993). 'The development of a new gravity-opportunity model for trip distribution', *Environment and Planning A*, 25, 817-826.

Roy, J. R. (1993). 'Comment on "The development of a new gravity-opportunity model for trip distribution"', *Environment and Planning A*, 25, 1689-1691.

Roy, J. R. (1997). 'A more consistent retail demand model : integration of discrete and continuous choice', *Geographical and Environmental Modelling*, 1, 83-96.

APPENDIX A

Key Results for Random Job Choice (β=0)

CASE 1A : [Also demonstrated in Anderson, Roy and Brotchie (1986)]

Given that both jobs and workers are distributed uniformly over a unit circle and that workers choose jobs randomly, find the average trip length t_u , assuming that workers travel on shortest path routes on the radial/circumferential transport network.

Let an incremental number ($r_j\, dr_j\, d\theta_j$) jobs be located at relative radius r_j at angle θ_j around from a datum. Then consider an incremental number of workers ($r_i\, dr_i\, d\theta_i$) be located at relative radius r_i at angle θ_i from the θ_j line and let these workers and jobs interact with each other via work travel. Then, the average travel distance t_0 can be evaluated via the expressions

$$t_u = (2/\pi^2) \int_0^{2\pi} d\theta_j \int_0^1 r_j\, dr_j \; \mathbf{\{} \int_0^2 [\int_0^{r_j} (r_i\, \theta_i + r_j - r_i\,)\, r_i\, dr_i + \int_{r_j}^1 (r_j\, \theta_i - r_j + r_i)\, r_i\, dr_i$$

$$+ \int_2^{\pi} d\theta_i \int_0^1 (r_i + r_j)\, r_i\, dr_i \; \mathbf{\}}$$

Within the **{ .... }** brackets, the first term corresponds to the case of making the circumferential part of the trip from the origin i, the second term denotes taking the circumferential part to the destination j and the third term occurs when the subtended angle between the i and j lines is greater than two radians and shortest path travel is purely radial, with the worker first travelling into the centre of the circle and then out to the job. The result comes out as

$$t_u = [\, 4 - 2/\pi] / 3 \qquad (\approx .9938)$$

Thus, the average distance travelled here is close to the unit radius of the circle.

CASE 2A :

Let workers be distributed radially over the unit circle according to the negative linear relationship [a+b(1-r_i)] where r_i is the relative worker radius at any point and let jobs also be distributed radially over the unit circle according to the negative linear relationship [c+d(1-r_j)] where r_j is the relative job radius at any point, adjusted such that the total number of workers equals the total number of jobs. Assuming that workers choose jobs randomly, find the average trip length t_v given that workers travel on shortest path routes on the radial/circumferential transport network.

This represents a generalization of Case 1A, allowing for probable increased residential density and increased employment density as the centre of the circular zone is approached. By an initial integration, it can be determined that the total number of workers is (a + b/3) and the total number of jobs (c + d/3), where c or d are chosen to ensure that these two terms are equal. Then, similarly as above, but allowing the individual worker and job increments to be weighted by their respective densities, the average travel distance t_v for this variable density problem is expressed as

$$t_v = [18/\{\pi(3a+b)(3c+d)\}] \int_0^{2\pi} d\theta_j \int_0^1 [c+d(1-r_j)]\, r_j\, dr_j \,\mathbf{\{} \int_0^2 \mathbf{[} \int_0^{r_j} (r_i\theta_i+r_j-r_i)\,[a+b(1-r_i)]\, r_i\, dr_i$$

$$+ \int_{r_j}^1 (r_j\theta_i-r_j+r_i)\,[a+b(1-r_i)]\, r_i\, dr_i \,\mathbf{]}\, d\theta_i + \int_2^\pi d\theta_i \int_0^1 (r_i+r_j)\,[a+b(1-r_i)]\, r_i\, dr_i \,\mathbf{\}}$$

After a considerable amount of quite tedious algebra, the result t_v comes out as

$$t_v = [1/[(3a+b)(3c+d)]\,\mathbf{\{}\,[24ac+7(ac+bd)+2bd]/2 - [336ac+91(ac+bd)+26bd]/35\pi\,\mathbf{\}}$$

For the uniform case, where a=c and b=d=0, it is simply confirmed that t_v comes out as the uniform result t_u given above in Case 1A. It is now straightforward to test alternative distributions, especially those cases where the densities of both workers and jobs increase in consonnance as one approaches the centre of the circle, that is, a=c and b=d=na where n≥0.

CASE 3A :

Let A be any point on the rim of the unit circle. Given that either workers are distributed uniformly ***or*** *that jobs are distributed uniformly over the unit circle, find the average travel distance t_r from A to any job* ***or*** *to A from any worker, given that travel occurs via shortest path routes on the radial/circumferential transport network.*

As the two cases give identical results, we confine ourselves to travel from A to any job. Let the incremental area (r dr dθ) contain jobs at radius r and angle θ from a datum. Then, the average distance per trip t_r from the rim point A is given as

$$t_r = (2/\pi)\ \mathbf{\{} \int^1 r\, dr\, [\int^2 \{ r\theta + (1-r) \}\, d\theta + \int^\pi \{r+1\}\, d\theta\,]\ \mathbf{\}}$$

0 0 2

where the first term in the [....] brackets represents travel from A by the shortest path radial/circumferential route to any job, and the second term denotes pure radial travel from A to the centre and from the centre out to the job. The result comes out as

$$t_r = (5 - 4/\pi) / 3 \qquad (\approx 1.242)$$

In practice, the above result is important for *interzonal* travel, where A represents the point either where one exits the zone via the transport network to reach the final destination zone, or where one enters the zone via the transport network from an adjacent zone en route from the origin zone.

APPENDIX B
Key Results for Travel-Distance-Minimizing Job Choice ($\beta \to \infty$)

CASE 1B : [Also shown by Bonsall(1975)]

If workers are distributed uniformly over the unit circle and all commute to the centre of the circle to work, determine the trip length t_m averaged over all workers.

Note that, as the workers here have no choice of job location, the result below is also valid for any value of β, including the previous value β=0. Using symmetry, it is clear that the average trip length t_m in terms of an increment ($r\, dr\, d\theta$) of workers is given as

$$t_m = (4/\pi) \int_0^{\pi/2} d\theta \int_0^1 r^2 \, dr = (2/3)$$

This is a standard result for intrazonal travel to the centre of the unit circle.

CASE 2B :

Let workers be distributed radially over the unit circle according to the negative linear relationship [a+b(1-r)] where r is the relative worker radius at any point. Then, if all workers commute to the centre to work, find the trip length t_w averaged over all workers.

Using integration, the total number of workers (and jobs) is given as [π(a+b/3)]. Then, as above, but using the density weighting [a+b(1-r)]], the result t_w comes out as

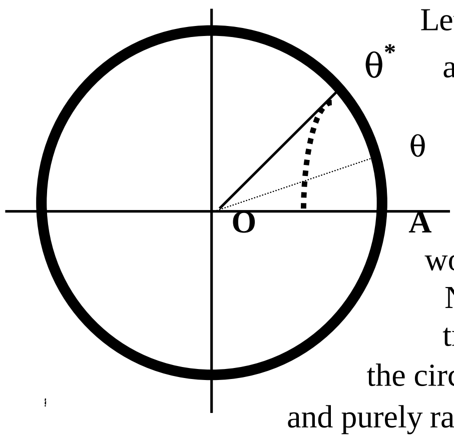
$$t_w = [4/\{\pi(a+b/3)\}] \int_0^{\pi/2} d\theta \int_0^1 [a+b(1-r)] \; r^2 \, dr = (4a+b)/[2(3a+b)]$$

This is seen to yield the result (2/3) as for the uniform density Case 1B above when the gradient b is set to zero. On the other hand when the outer density a approaches zero, t_w approaches its minimum value 0.5.

CASE 3B* :

Let A be any point on the rim of the unit circle. The workers are assumed to be distributed uniformly over the circle, with a ratio p of them commuting to A to obtain jobs outside their own zone, and the remainder (1-p) obtaining the (1-p) jobs at the centre of the circle. Find the average trip distance t_c for the central (intrazonal) workers and the average exit distance t_x for the external workers leaving from A, assuming that the total travel distance of all workers is to be minimized.

* The author is most grateful to his former CSIRO colleague, Joe Flood, for collaboration on this proof.



The situation is illustrated in the Figure below. Consider that the bold dotted arc includes all the commuters 'πp' who work outside the zone and exit at A. This leaves area π(1-p) to commute to the centre O of the circle. Let θ^* be the angle subtended at the rim of the circle at the boundary between the two areas and let θ be the angle subtended at any intermediate point. The first step is to determine the function **R(θ)** of the boundary line separating the two areas, such that the average travel distance of all workers who are resident in the circle is minimized. Note that, whereas all workers who commute to O travel radially, those who exit at A to jobs outside the circle travel circumferentially and radially for $0 \le \theta < 2$ and purely radially for $2 \le \theta \le \pi$, where they proceed to A via O. Thus, the total travel distance D is minimized in terms of **R(θ)**, constrained by the requirement to have (πp) workers travelling to A, in the form

(i) $0 \le \theta^* < 2$

$$D = \min_{\mathbf{R(\theta)}} \int_{\theta^*}^{\pi} d\theta \int_0^1 r^2\,dr + \int_0^{\theta^*} d\theta \int_0^{\mathbf{R(\theta)}} r^2\,dr + \int_0^{\theta^*} d\theta \int_{\mathbf{R(\theta)}}^1 [r\theta + (1-r)]\,r\,dr$$

$$-\,\lambda\,\left[\,\pi p/2 - \int_0^{\theta^*} d\theta \int_{\mathbf{R(\theta)}}^1 r\,dr\,\right]$$

where λ is the Lagrange multiplier on the area constraint for workers with external jobs. In the integrand, the first term is the total travel distance of internal workers resident in the sector θ^* to π, the second term represents the travel of the rest of these workers between the θ^* line and the **R(θ)** curve and the last term is the total travel to A of the external workers bounded by the circle and the **R(θ)** curve. In this problem without **R(θ)** derivatives, the variational calculus is not required. Realizing that only those integrals with **R(θ)** limits contribute to the maximization process, and reversing the sign on terms where $\mathbf{R(\theta)}$ occurs as a lower limit, the extremum occurs for zero value of the total integrand, yielding

$$\mathbf{R(\theta)}^2 - [\, \mathbf{R(\theta)}\, \theta + (1 - \mathbf{R(\theta)})\,]\, \mathbf{R(\theta)} - \lambda\, \mathbf{R(\theta)} = 0$$

This simplifies to

$$\mathbf{R(\theta)} = (1 + \lambda) / (2 - \theta)$$

The unknown multiplier λ can be eliminated from the above by substitution of $\mathbf{R(\theta)}$ above into the constraint equation. Knowing that θ^* occurs when $\mathbf{R(\theta)}$ is unity, θ^* comes out as

$$\theta^* = (1 - \lambda)$$

allowing the constraint integral to be evaluated, yielding λ and thus θ^* as

$$\lambda = 1 - \sqrt{2\pi p} \ ; \quad \theta^* = \sqrt{2\pi p}$$

giving $\mathbf{R(\theta)}$ in its final explicit form

$$\mathbf{R(\theta)} = (2 - \theta^*) / (2 - \theta) = (2 - \sqrt{2\pi p}) / (2 - \theta)$$

As on the horizontal axis where θ=0, $\mathbf{R(\theta)}$ is seen to be $(2 - \theta^*)/2$, the internal boundary of the area enclosing the external workers flattens to a straight vertical line at a value of θ^* yielding $(2-\theta^*)/2 = \cos\theta^*$, that is $\theta^*=1.110$ radians. Thereafter, it reverses curvature in respect to that illustrated above, finally becoming the radial straight lines $\theta^*=2$ and $\theta^*=2\pi-2$ at the boundaries of the feasible area.

Given both $\mathbf{R(\theta)}$ and θ^*, it is possible to evaluate the total travel T_A to A as the third term in the expression for D above, yielding T_A and the average distance t_x as

$$T_A = \theta^{*3} / 4 \qquad \text{and} \qquad t_x = [\sqrt{2\pi p}]^3 / [4\pi p] = (\sqrt{2\pi p})/2$$

Similarly, the first and second terms in D can be summed to yield the total travel T_O of the internal commuters and their average trip distance t_c as

$$T_O = (2\pi/3)\,[\,1 - 3p/2 + (p\,\sqrt{2\pi p})/4\,] \quad \text{and} \quad t_c = (2/3)\,[\,1 - p(2-\theta^*)/\{4(1-p)\}\,]$$

As expected, t_c comes out as the classical result (2/3) at $\theta^*=2$, where the $\mathbf{R(\theta)}$ curve is a straight radial line and we just have the geometry of sectors. At all other values of $\theta^*<2$, t_c is less than (2/3) because some outer commuters go to A rather than to O.

(ii) $2 \le \theta^* \le \pi$

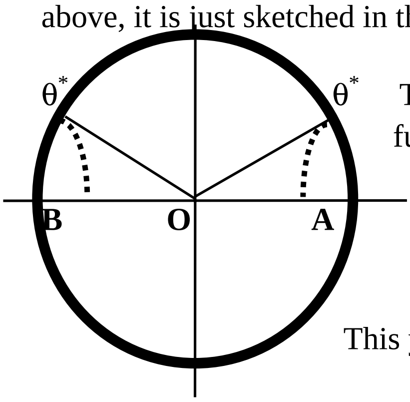
It is easy to show for this range of θ^* that the boundary line between the external and internal workers remains a straight radial line, as for $\theta^*=2$ above. Then, simple integration over the sectors yields the following results

$t_x = 2(2\pi p - 1)/[3\pi p]$ and $t_c = (2/3)$

CASE 4B :

Let A and B be two points in a straight line on opposite rims of the unit circle. The workers are assumed to be distributed uniformly over the circle, with ratios (p/2) of them commuting to both A and B to obtain jobs outside their own zone, and the remainder (1-p) obtaining the (1-p) jobs at the centre of the circle. Find the average trip distance t_c *for the central (intrazonal) workers and the average exit distance* t_x *for the external workers leaving from A or B, assuming that the total travel distance of all workers is to be minimized.*

This problem, illustrated below, is a variation on the preceding, where the commuters now exit in opposite directions to external jobs. As the proof is closely related to that above, it is just sketched in the following.



The only expression which changes in the objective function above is in the λ constraint, which becomes

$$- \lambda \left[\pi p/4 - \int_0^{\theta^*} d\theta \int_{\mathbf{R}(\theta)}^{1} r dr \right]$$

This yields $\mathbf{R}(\theta)=(2-\theta^*)/(2-\theta)$ as before, but now θ^* is given as

$$\theta^* = \sqrt{\pi p}$$

This leads to the average trip distance for external trips, to leave at either A or B, as

$$t_x = \theta^* / 2 = (\sqrt{\pi p}) / 2$$

The expression for the average internal trip distance t_a is identical to that for Case 3B, except for the different definition of θ^*. Note that, even at $\theta^*=\pi/2$, p is not yet unity, but equal to $(\pi/4)$, with the remainder of the area being filled as θ^* approaches 2.